

2

$$\begin{bmatrix} 0 & t \\ t & \Delta \end{bmatrix} \{ \varphi_1, \varphi_2 \} \rightarrow \begin{bmatrix} 0 & -t \\ -t & \Delta \end{bmatrix} \{ \varphi_1, -\varphi_2 \} \quad t \leftrightarrow -t : \text{the same}$$

$$\begin{bmatrix} 0 \\ t & \Delta_1 \\ t & t & \Delta_2 \end{bmatrix} \{ \varphi_1, \varphi_2, \varphi_3 \} \rightarrow \begin{bmatrix} 0 \\ -t & \Delta_1 \\ t & -t & \Delta_2 \end{bmatrix} \{ \varphi_1, -\varphi_2, \varphi_3 \}$$

$$\rightarrow \begin{bmatrix} 0 \\ t & \Delta_1 \\ -t & -t & \Delta_2 \end{bmatrix} \{ \varphi_1, \varphi_2, -\varphi_3 \}$$

$$\rightarrow \begin{bmatrix} 0 \\ -t & \Delta_1 \\ -t & t & \Delta_2 \end{bmatrix} \{ \varphi_1, -\varphi_2, -\varphi_3 \}$$

$$\rightarrow \neq \begin{bmatrix} 0 \\ -t & \Delta_1 \\ -t & -t & \Delta_2 \end{bmatrix} \text{ different!} \quad t \leftrightarrow -t \text{ different!}$$

$$\begin{bmatrix} 0 & & \\ t & 0 & \\ t & t & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = E \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \rightarrow \begin{vmatrix} -E & t & t \\ t & -E & t \\ t & t & -E \end{vmatrix} = 0 \rightarrow$$

$$\begin{aligned} -E(E^2 - t^2) - t(-tE - t^2) + t(t^2 + Et) &= 0 \rightarrow \\ -E^3 + E(t^2 + t^2 + t^2) + t^3 + t^3 &= 0 \rightarrow E^3 - 3t^2E - 2t^3 = 0 \\ (E+t)(E^2 - 2t^2) + 2t^2E - tE^2 - 3t^3E &= 0 \rightarrow (E+t)(E^2 - 2t^2) - t^2E - tE^2 = 0 \rightarrow \\ (E+t)(E^2 - 2t^2) - Et(t+E) &= 0 \rightarrow (E+t)(E^2 - Et - 2t^2) = 0 \rightarrow \\ (E+t)(E+t)(E-2t) &= 0 \rightarrow E_1 = -t, E_2 = -t, E_3 = 2t \end{aligned}$$

$$t = \text{positive} \quad \begin{array}{c} \downarrow 2|t \\ 0 \\ \downarrow |t \end{array} \quad t = \text{negative} \quad \begin{array}{c} \overline{\downarrow |t} \\ 0 \\ \downarrow 2|t \end{array}$$