Bloch Theorem :

$\psi_{\underline{k}}(r) = u_{\underline{k}}(r) \cdot e^{i\underline{k}\underline{r}}$ [see Kittel]

with $u_{\underline{k}}(r) = u_{\underline{k}}(\underline{r}+\underline{T})$ and $\underline{T}$ is a lattice vector.

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localized basis $\varphi(\underline{r}-\underline{R}_i)$; i.e localized around site $\underline{R}_i$

construct $\psi_{\underline{k}}(r)$ with

$$\psi_{\underline{k}}(r) = \frac{1}{\sqrt{N}} \sum_{\underline{R}_i} e^{i\underline{k}\underline{R}_i} \varphi(\underline{r}-\underline{R}_i)$$

with $\underline{k}$ being a reciprocal lattice vector.

N = number of sites

We can show that this $\psi_{\underline{k}}(r)$ is a Bloch function:

$$\psi_{\underline{k}}(r) = e^{i\underline{k}\underline{r}} \cdot e^{-i\underline{k}\cdot\underline{r}} \frac{1}{\sqrt{N}} \sum_{\underline{R}_i} e^{i\underline{k}\underline{R}_i} \varphi(\underline{r}-\underline{R}_i)$$

$$= e^{i\underline{k}\cdot\underline{r}} \cdot \frac{1}{\sqrt{N}} \sum_{\underline{R}_i} e^{-i\underline{k}(\underline{r}-\underline{R}_i)} \varphi(r-R_i)$$

$$\equiv e^{i\underline{k}\underline{r}} \cdot u_{\underline{k}}(r)$$ with

$$u_{\underline{k}}(r) \equiv \frac{1}{\sqrt{N}} \sum_{\underline{R}_i} e^{-i\underline{k}(\underline{r}-\underline{R}_i)} \varphi(\underline{r}-\underline{R}_i)$$

$\downarrow$

this is no longer a function of $\underline{R}_i$ and is insensitive to $\underline{r} \rightarrow \underline{r}+\underline{T}$ substitution

i.e. $u_{\underline{k}}(r) = u_{\underline{k}}(\underline{r}+\underline{T})$.